Simplified Lattice Beam Elements for Geometrically Nonlinear Static, Dynamic, and Postbuckling Analysis

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Abstract

A SIMPLE beam finite element for modeling the geometrically nonlinear behavior of flexible space lattice beams with repetitive geometry is developed. The element is formulated using expressions for the strain and kinetic energies of an equivalent beam continuum. To account for the nonlinear effects, the incremental stiffness matrix of the equivalent continuum is used. To evaluate the present nonlinear formulation and assess its accuracy, a series of three nonlinear beam problems are solved using this simple element: 1) static large-deflection analysis, 2) large-amplitude free-vibration analysis, and 3) buckling and postbuckling analysis. Alternative solutions for the three examples show that the present formulation may be used as an efficient substitute for a detailed truss model in static and dynamic geometrically nonlinear analysis of beamlike lattice structures.

Contents

Beamlike lattice structures have attracted considerable attention recently for their potential use in space applications. Unfortunately, a complete and detailed finite element approach to the structural modeling problem may be too time-consuming and costly due to the large number of elements and nodes within the structure, especially during the initial design phases, when the structure and its associated systems may be subject to major change. The problem is compounded if structural nonlinearities are considered.

Several methods of simplification have been proposed to overcome the complexity of this modeling problem. Of these methods, the replacement of the lattice by an energy equivalent continuum seems very promising. (See, e.g., Ref. 1). In Ref. 2, Berry et al. presented a formulation of simplified "equivalent finite element" for use in the linear modeling of the "equivalent continuum" and showed that these simple finite element models can be used economically and accurately in a control law design problem. The present study presents an extension of the basic simple element formulation to include geometrically nonlinear effects. This nonlinear extension allows the static and dynamic large-deflection behavior as well as the global buckling and postbuckling response of beamlike lattice structures to be analyzed using the simplified finite element models.

One particular lattice beam geometry is considered in this study. It has ten repeating cells of the "single-bay, double-lace" type.[3] The simplified finite element model is a standard beam model of the equivalent continuum. The linear element stiffness matrix and consistent mass matrices were formulated in Ref. 2. The present approach to the geometrically nonlinear analysis is simply to use the incremental stiffness matrix of the equivalent continuum. For the example problems examined in this study, the explicit form of this matrix is given in Ref. 4. For more general three-dimensional problems, the results of Ref. 5 may be used. This matrix requires an expression for the axial force P, which is given by

$$P=\overline{AE}[e+\tfrac{1}{2}(\theta_y)^2+\tfrac{1}{2}(\theta_z)^2] \tag{1}$$

where $\overline{AE}$ is the equivalent axial stiffness, e is strain due to extension along the element axis, and $\tfrac{1}{2}(\theta_y)^2$ and $\tfrac{1}{2}(\theta_z)^2$ are strains due to large rotations. These terms are functions of the displacement variables of the equivalent beam continuum. The incremental equations of motion for the simplified finite element model for the large-deflection and postbuckling analyses are solved by an incremental load procedure, together with a Newton-Raphson iterative method to assure convergence to the true equilibrium state. The nonlinear free-vibration problem is solved using an iterative procedure beginning with a suitably scaled small-amplitude vibration mode shape.

To evaluate the accuracy of the above formulation, a detailed finite element model of the lattice beam is produced by modeling each lattice member as an axial force bar element. By using a strain-displacement relation of the following form,

$$\epsilon \simeq e+\tfrac{1}{2}\phi_y^2+\tfrac{1}{2}\phi_z^2+\tfrac{1}{2}e^2 \tag{2}$$

large-deflection geometrically nonlinear behavior may be predicted. The present simplified model accounts for nonlinearity globally. Local nonlinear effects may be accounted for in the detailed complex models by using, for example, the techniques of Ref. 6. Therefore, it is assumed that each bar remains linearly elastic, straight, and stable during all truss motions. The static problems are again solved using an incremental load and equilibrium iteration procedure, and the large-amplitude free-vibration problem is solved as above, using an iterative procedure based on mode shape. Details may be found in Ref. 3.

Results

To illustrate the present formulation, three example problems were solved using both the simplified and the detailed models: 1) static large-deflection analysis, 2) large-amplitude free-vibration analysis, and 3) buckling and postbuckling analysis. In the static large-deflection analysis, the example considered is a cantilever beam under a tip load. Results are presented in Fig. 1. It is seen that the accuracy of the simple model increases with the number of elements in the model but is approximately converged with four elements. Figure 1 shows that the nonlinear load-deflection curve of the simple finite element models agrees closely with that of the detailed model.

The sample large-amplitude vibration analyses are of a simply supported beam with immovable in-plane end restraints and of a clamped-clamped beam. Results are shown in Fig. 2, which presents amplitude vs fundamental frequency data for the two cases. Although 1-10 elements were used in the simple model, the frequencies were con-

Received March 4, 1985; presented as Paper 85-0694 at the AIAA/ASME/ASCE/AHS 26th Structures, Structural Dynamics and Materials Conference, Orlando, FL, April 15-17, 1985; synoptic submitted Jan. 7, 1986. Full paper available from AIAA Library, 555 W. 57th Street, New York, NY 10019. Price: microfiche $4.00; hard copy, $9.00. **Remittance must accompany order.**

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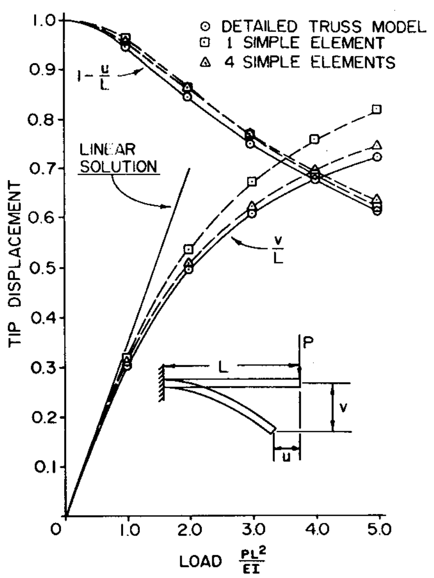

Fig. 1 Load deflection curves: large static deflection.

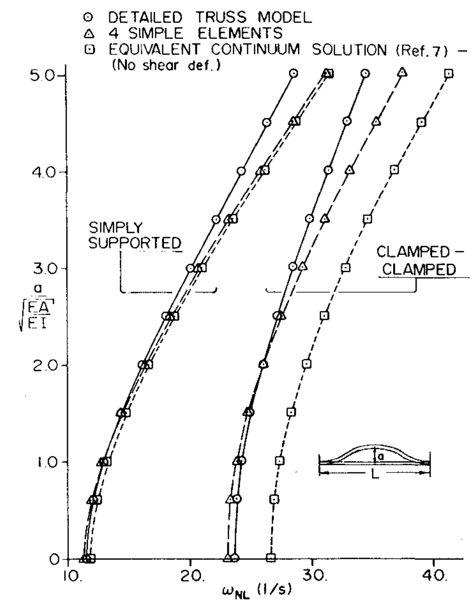

Fig. 2 Natural frequencies: large-amplitude vibration.

sidered to be converged with four elements. From Fig. 2, it is seen that the frequencies increase with amplitude due to the nonlinear stiffening effect of the axial forces induced by a stretching of the neutral axis. Although shear deformation and rotatory inertia do not appear to influence the shape of the curves, they do have an effect on the magnitudes of the frequencies, especially for the clamped-clamped case. It is seen that the agreement between the results of the simple finite element model and those of the detailed model is good at low amplitudes.

The buckling and postbuckling analysis is of a column under a compressive axial tip load. The buckling load for the detailed model is computed incrementally and is obtained when the fundamental eigenvalue of the total stiffness matrix is zero. However, the buckling load for the simplified model is calculated by an eigenvalue procedure. The postbuckling analysis for both models is approached in the same manner. A very small multiple of the fundamental eigenvector associated with the Euler buckling load is used to begin the postbuckling analysis. The load-displacement curve is followed by the use of an incremental load, equilibrium iteration procedure. The buckling loads and postbuckling load-displacement paths are shown in Fig. 3. The buckling loads dif-

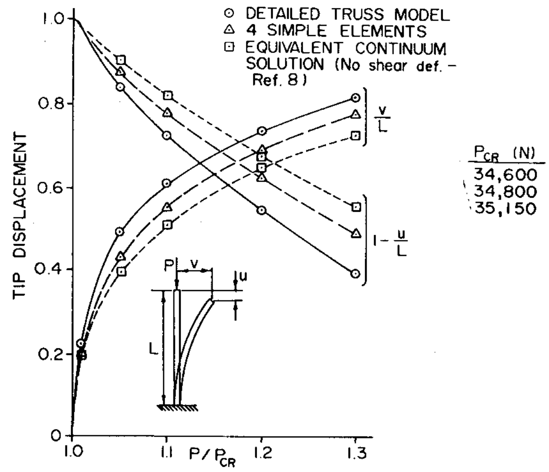

Fig. 3 Load deflection curves: postbuckling analysis.

fer by less than 2%. Furthermore, the results of the simplified finite element models follow the same trend as those of the detailed model with some discrepancies. For this particular analysis, the effects of shear deformation are significant.

Conclusions

A simplified beam finite element has been developed for use in geometrically nonlinear analysis of beamlike lattice structures. The element has been used in a series of three typical nonlinear structural analysis: 1) static large-deflection analysis, 2) large-amplitude free-vibration analysis, and 3) buckling and postbuckling analysis. Results show that the approach to the geometrically nonlinear problem presented here is valid. It is suggested that the more economical simplified finite element model may be used in place of the complicated detailed model without appreciable loss of accuracy, especially during the early states of the design process.

Acknowledgment

This work was sponsored under Air Force Office of Scientific Research Contract AFOSR-83-0104 administrated by Dr. Anthony Amos.

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